Name:	
Class:	

JAMES RUSE A H S
GESTA NON VERBA

YEAR 12

ASSESSMENT TEST 1
TERM 4, 2014

MATHEMATICS

Time Allowed – 90 Minutes
(Plus 5 minutes Reading Time)

Question	1	2	3	4	Total
Integration	a,b /7	a,b /7	b,c,d /10	a,b /8	/32
Coord Geometry	c /8		a /5		/13
Geometry		c /8		c /7	/15
Total	/15	/15	/15	/15	/60

- *All* questions may be attempted.
- *All* questions are of equal value.
- Department of Education approved calculators and templates are permitted.
- In every Question, show all relevant mathematical reasoning and/or calculations.
- Marks may not be awarded for careless or badly arranged work.
- No grid paper is to be used unless provided with the examination paper.

The answers to all questions are to be returned in separate bundles clearly labeled Question 1, Question 2, Question 3, Question 4.

Each question must show (in the top right hand corner) your Candidate Number.

QUESTION 1 (15 Marks)

Marks

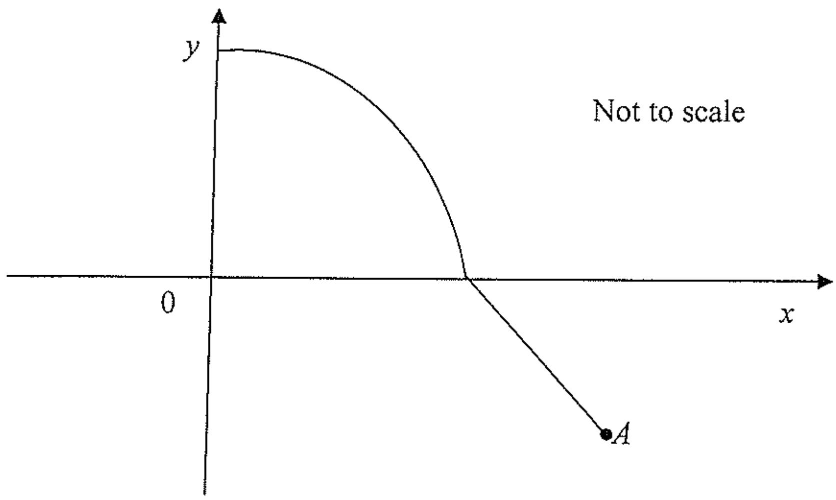
(a) Find the following indefinite integrals

(i) $\int (1+x^2)\,dx$ **1**

(ii) $\int (1+\tan^2 x)\,dx$ **2**

(b) The diagram below shows the graph of the function $f(x)$ for $0 \le x \le 3$ where:

$$f(x) = \begin{cases} \sqrt{4-x^2} & \text{for } 0 \le x \le 2 \\ 4-2x & \text{for } 2 \le x \le 3 \end{cases}$$



(i) Find the coordinates of point A. **1**

(ii) Evaluate the integral $\int_0^3 f(x)\,dx$. **3**

(c) (i) Plot the points $A(-2, 2)$, $B(-1, -5)$ and $C(6, -6)$ on a number plane. **1**

(ii) Point P is the midpoint of AC. Write down the coordinates of P. **1**

(iii) Find the gradient of BP. **1**

(iv) Show that BP is perpendicular to AC. **1**

(v) Find the equation of the line BP. . **1**

(vi) Point P is the midpoint of BD. Find the coordinates of D. **1**

(vii) Is the quadrilateral $ABCD$ a square? Justify your answer. **2**

QUESTION 2 (15 Marks) START A NEW PAGE

(a) Evaluate the definite integrals. Give your answers in simplified exact form:

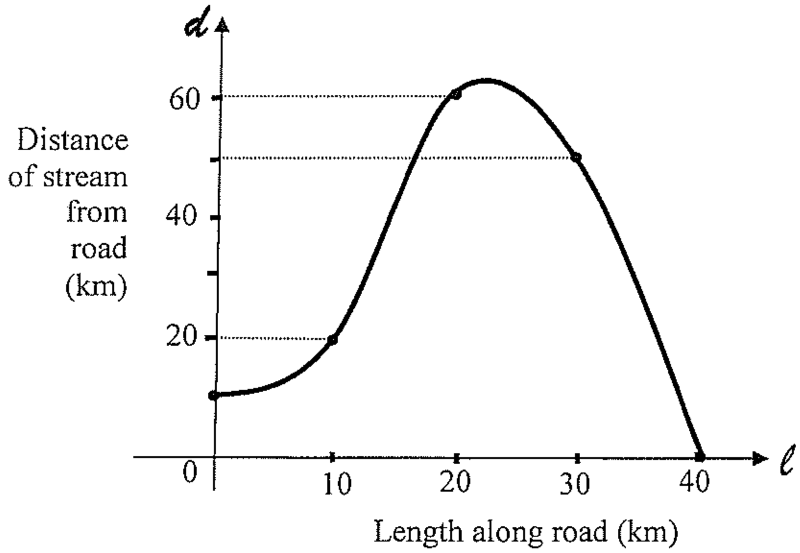
(i) $\int_1^2 \frac{x^2+4x}{x^3}\,dx$ **2**

(ii) $\int_0^2 e^{1-4x}\,dx$ **2**

Question 2 continued over the page.

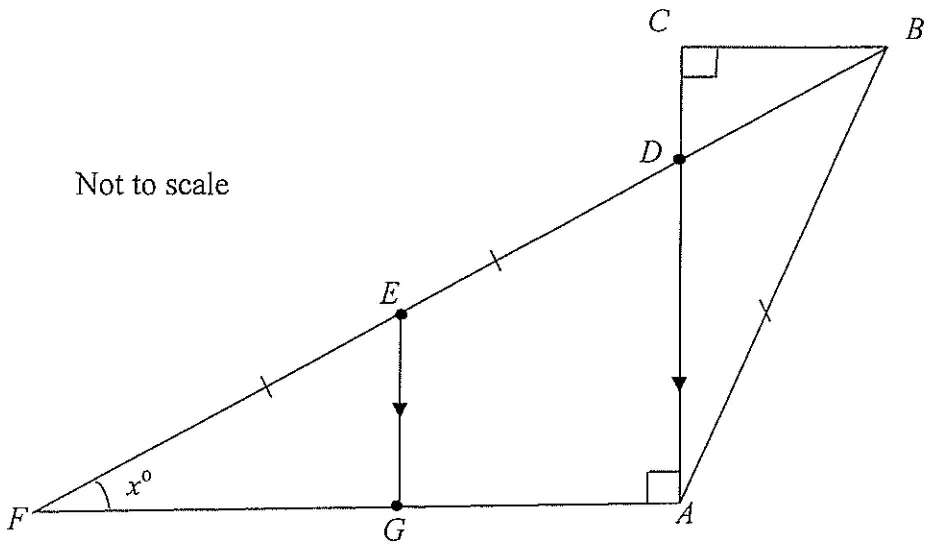
Question 2 continued **Marks**

(b) The graph below shows the distance of a stream from a particular length of road.



Use Simpson's Rule with 5 function values to estimate the area of land between stream and the 40 km section of road. **3**

(c) In the diagram below $\angle BCA = \angle CAF = 90°$, $EG // CA$ and $AB = DE = EF$.



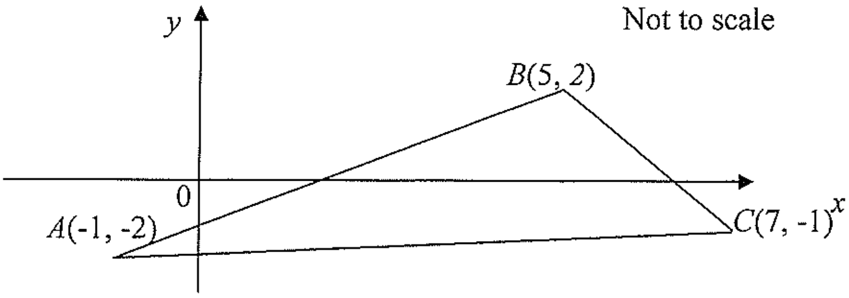
(i) Copy the diagram and show that $\angle CBD = \angle AFD$, giving reasons. **2**
Let $\angle AFD = x°$.

(ii) Prove that $\Delta EGA \equiv \Delta EGF$ **3**

(iii) Explain why $\angle ABD = 2 \times \angle CBD$. **3**

QUESTION 3 (15 Marks) START A NEW PAGE **Marks**

(a) A triangle is formed by joining the points $A(-1, -2)$, $B(5, 2)$ and $C(7, -1)$ on the number plane as shown in the diagram below.



(i) Find the length of AB. **1**

(ii) Find the equation of AB. **2**

(iii) Find the area of ΔABC. **2**

(b) Find the equation of the curve that passes through the point $A(1, 2)$ for which $\dfrac{dy}{dx} = \dfrac{2}{x\sqrt{x}}$. **3**

(c) Copy and complete the table below using exact values for the function

$$f(x) = \sqrt{\sin x}\,.$$

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$
$f(x) = \sqrt{\sin x}$					

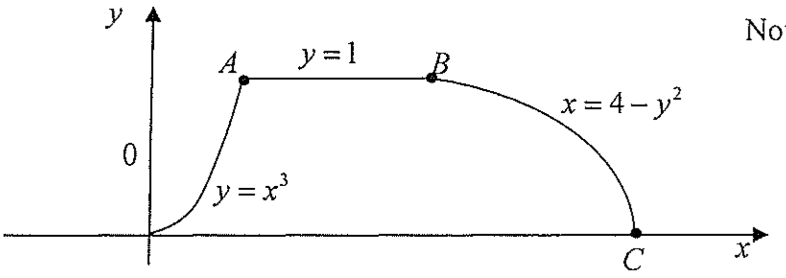
Use the trapezoidal rule with 5 function values to find an approximate value for the integral **4**

$$I = \int_0^{\frac{2\pi}{3}} \sqrt{\sin x}\, dx\,.$$ Give your answer to 2 decimal places.

Question 3 continued over the page.

(d) A garden bed is designed such that it is bounded by the functions:

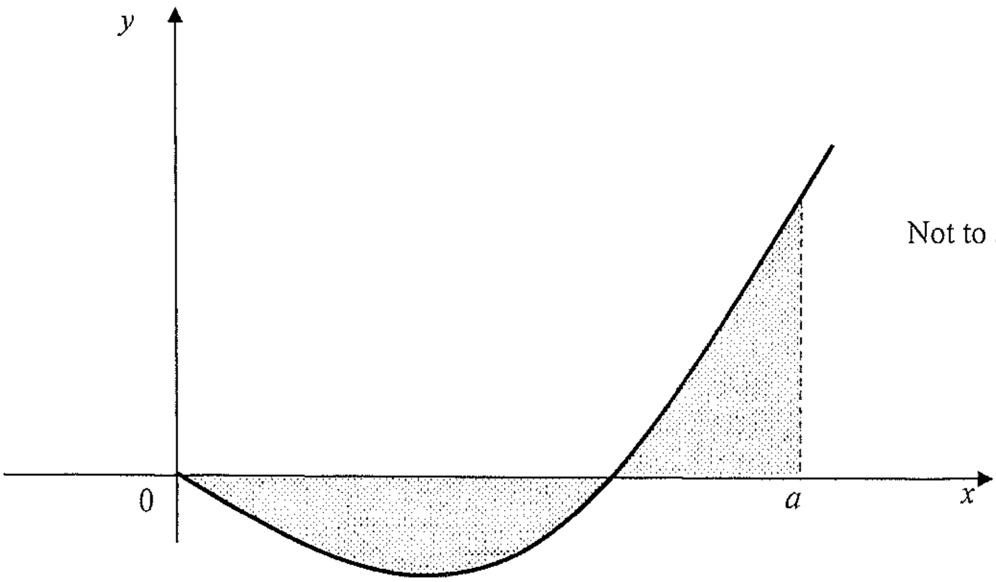
$x = 4 - y^2$, $y = x^3$, $y = 1$ and $y = 0$, as shown in the diagram below.



Calculate the exact area of the garden bed. **3**

QUESTION 4 (15 Marks) START A NEW PAGE

(a) The diagram shows the graph of the function $y = x(x^2 - 2)$ for $x \geq 0$.



Find the value of a, for $a > 0$, so that the two shaded regions have equal area. **3**

Question 4 continued over the page.

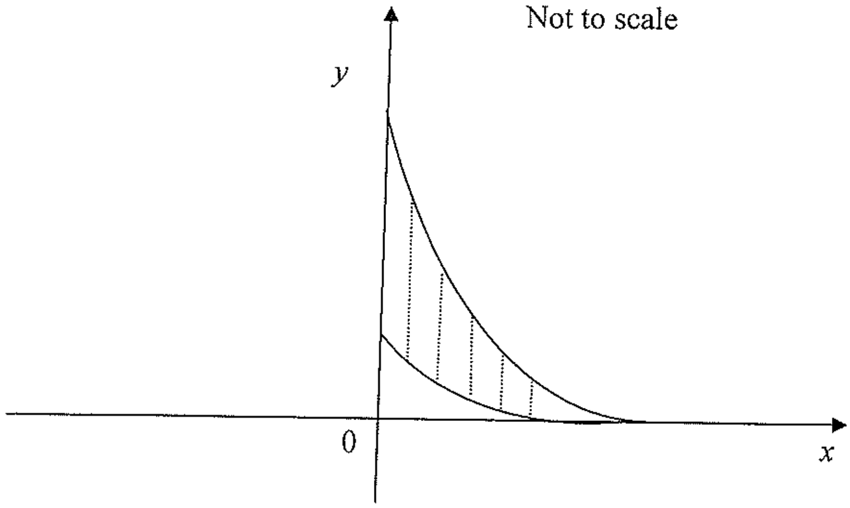
Question 4 continued

Marks

(b) The diagram below shows the graphs of the functions:

$$4y=(x-1)^2 \text{ and } y=(x-1)^2 \text{ for } 0\le x\le 1.$$

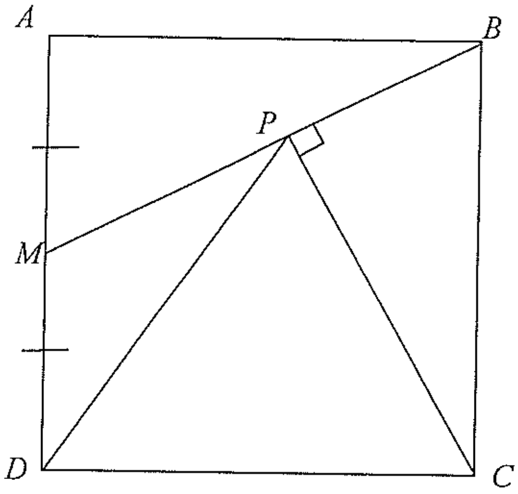
Not to scale



The area between these graphs is rotated through 360° about the y axis. **5**

Find the volume of the resulting solid.

(c) $ABCD$ is a square of side length 5cm. M is the midpoint of AD. CP is drawn perpendicular to BM.



Not to scale

(i) Copy the diagram and prove that ΔABM is similar to ΔPCB. **2**
(ii) Find the exact length of CP, giving reasons. **2**
(iii) What type of triangle is ΔCDP? Justify your answer. **3**

End of Examination

2 UNIT SOLUTIONS

(1) a) i) $\int 1+x^2\,dx = x+\frac{x^3}{3}+k$ (1)

ii) $\int \sec^2 x\,dx = \tan x + k$ (1) (1) ← must have constant

b) i) A is $(3,-2)$ (1)

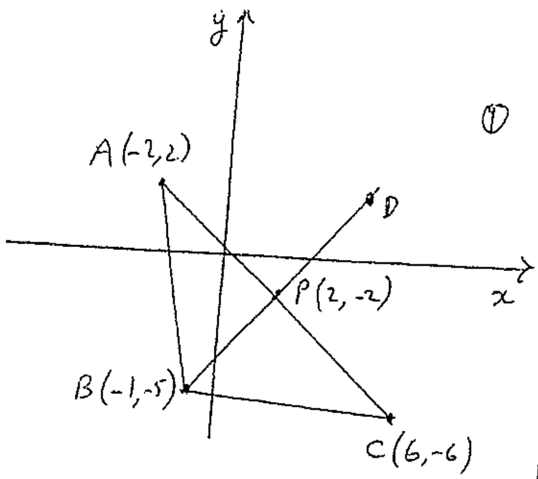
ii) $\int_0^2 f(x)\,dx = \frac{1}{4}\pi 2^2 = \pi$ (1)

(quadrant of circle)

$\int_2^3 f(x)\,dx = -1$ ← (1) or Area $=1$

$\therefore \int_0^3 f(x)\,dx = \pi - 1$ (1)

c) i)



(1)

ii) P is $\left(\frac{6-2}{2}, \frac{-6+2}{2}\right) = (2,-2)$ (1)

iii) $m_{BP} = \frac{-2+5}{2--1} = 1$ (1)

iv) $m_{AC} = \frac{-6-2}{6--2} = -1$

$m_{AC} \cdot m_{BP} = 1\times -1 = -1$

$\therefore AC \perp BP$ (1)

v) $y+2 = 1(x-2)$

$y = x-4$ (1)

Any form

vi) D is $(2+3, -2+3) = (5,1)$ (1)

vii) NO. ABCD is not a square

$m_{AB} = \frac{2+5}{-2+1} = -7$

$m_{BC} = \frac{-6+5}{6+1} = -\frac{1}{7}$

$m_{AB}\cdot m_{BC} \neq -1$.

$\therefore$ AB not $\perp$ to BC.

My methods. (1) Calculations (1) Reason

(2) a) i) $\int_1^2 \frac{1}{x} + \frac{4}{x^2}\,dx$

$= \left[\ln x - \frac{4}{x}\right]_1^2$ (1)

$= \ln 2 - 2 - \ln 1 + 4$ (1)

$= 2 + \ln 2$ (1)

ii) $\int_0^2 e^{1-4x}\,dx = \left[\frac{e^{1-4x}}{-4}\right]_0^2$ (1)

$= \frac{e^{-7} - e}{-4} = \frac{e^1 - e^{-7}}{4}$

$= \frac{e^8 - 1}{4e^7}$ (1) Any form

b) (1)

$\frac{10}{3}\{10 + 4\times 20 + 60\}$

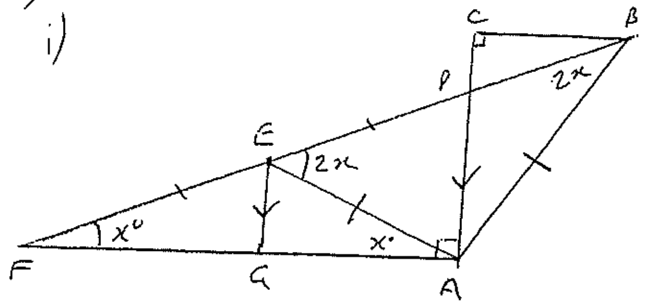
$+ \frac{10}{3}\{60 + 4\times 50 + 0\}$ (1)

$= \frac{10}{3}\{150 + 260\} = \frac{4100}{3}$

Area $\doteqdot 1366\frac{2}{3}\ \text{km}^2$ (1)

c)

i)



$\angle BCA = \angle CAF \ (= 90°)$

$\therefore$ BC // FA (alternate angles are equal) (1)

$\therefore \underline{\angle CBD = \angle AFD = x°}$ (BC//FA, alternate angles equal) (1)

ii) In Δs EGF, EGA

$\angle EGA + \angle DAG = 180°$ (DA // EG and cointerior angles add to 180°)

$\therefore \angle EGA = 90°$ ($\angle DAG = 90°$)

$\therefore \angle FGE = 90°$ (Straight angle) (1)

$\therefore \underline{\angle EGA = \angle FGE}$

Line EG is common

FE = ED (Given) and EG // DA.

FG = GA (A line parallel to one side of a triangle divides the other two sides in the same ratio). (1)

$\therefore \underline{\Delta EGA \equiv \Delta EGF \ (SAS)}$ (1)

Other methods possible

iii) $\because$ EA = EF (= ED = AB) (Corresponding sides in congruent triangles are equal)

$\angle EAF = \angle EFA = x$ (1)

(Equal angles opposite equal sides)

$\therefore \angle DEA = 2x$ (1)

(Exterior angle of triangle adds to sum of two opposite interior angles)

$\therefore \angle ABD = 2x$ (1)

(Equal angles opposite equal sides in ΔEBA)

$\therefore \underline{\angle ABD = 2\angle CBD}$ (= 2x)

(3) a) i) $AB = \sqrt{4^2 + 6^2}$

$= \sqrt{52} = \underline{2\sqrt{13} \text{ units}}$ (1)

ii) Two point form or similar

$\dfrac{y-2}{-2-2} = \dfrac{x-5}{-1-5}$ (1) gradient

$6(y-2) = 4(x-5)$

$3y - 6 = 2x - 10$

$\underline{2x - 3y - 4 = 0}$ } any form (1)

iii) $\perp$ distance C to AB is

$d = \left| \dfrac{2\times 7 - 3\times{}^{-}1 - 4}{\sqrt{2^2 + 3^2}} \right|$

$= \dfrac{13}{\sqrt{13}} = \sqrt{13}$. (1)

$\therefore$ Area $= \frac{1}{2}$ base $\times$ ht

$= \frac{1}{2} \, 2\sqrt{13} \times \sqrt{13}$

$= \underline{13 \ u^2}$ (1)

b) $\dfrac{dy}{dx} = \dfrac{2}{x^{3/2}}$ (1)

$y = -\dfrac{2}{x^{1/2}} \times 2 + k = \dfrac{-4}{\sqrt{x}} + k$ (1)

When $x = 1$, $y = 2$ $\therefore k = 6$.

Curve is $\underline{y = -\dfrac{4}{\sqrt{x}} + 6}$ (1)

c) (2)

x	0	$\pi/6$	$\pi/3$	$\pi/2$	$2\pi/3$
$\sqrt{\sin x}$	0	$\frac{1}{\sqrt{2}}$	$\sqrt{\frac{\sqrt{3}}{2}}$	1	$\sqrt{\frac{\sqrt{3}}{2}}$

(1) (1)

$f(x) \doteq \dfrac{\pi}{12}\left\{0 + \dfrac{2}{\sqrt{2}} + 2\sqrt{\dfrac{\sqrt{3}}{2}} + 2 + \sqrt{\dfrac{\sqrt{3}}{2}}\right\} = \underline{1.62}$ (2 DP) includes dp.

3 d) Area $= \int_0^1 x^3\,dx + 1\times 2 + \int_3^4 \sqrt{4-x}\,dx$

$= \left[\frac{x^4}{4}\right]_0^1 + 2 + \left[-\frac{2}{3}(4-x)^{3/2}\right]_3^4$ (1) (1) (1)

$= 2\tfrac{1}{4} + \frac{2}{3} = 2\tfrac{11}{12}\ u^2$

(4) a) $\int_0^a x(x^2-2)\,dx = 0$ (1)

$\int_0^a x^3 - 2x\,dx = \left[\frac{x^4}{4} - x^2\right]_0^a = 0$

$\therefore a^4 = 4a^2$ (1)

$a \neq 0 \therefore a^2 = 4$

$a = 2 \quad (a>0)$ (1)

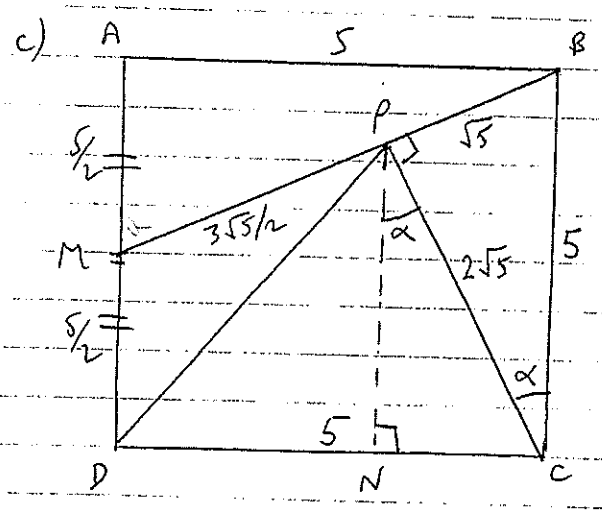
b) $V\Omega = \pi\int_0^1 x^2\,dy - \pi\int_0^{1/4} x^2\,dy$ (1)

$= \pi\left\{\int_0^1 (1-\sqrt{y})^2\,dy - \int_0^{1/4} (1-2\sqrt{y})^2\,dy\right\}$ (1) (1)

$= \pi\left\{\left[y + \frac{y^2}{2} - \frac{4y^{3/2}}{3}\right]_0^1 - \left[y + 2y^2 - \frac{8}{3}y^{3/2}\right]_0^{1/4}\right\}$ (1) (1)

$= \pi\left(\frac{1}{6} - \frac{1}{24}\right) = \frac{\pi}{8}\ u^3$ (1)

c)



i) In Δs ABM and PCB

$\angle A = 90°$ (corner of square)

$\therefore \angle MAB = \angle BPC$ (1)

$\angle AMB = 90° - \angle ABM$ (Angles of ΔABM add to 180°)

$\angle PBC = 90° - \angle ABM$ (Square corner at B adds to 90°)

$\therefore \angle AMB = \angle PBC$ (1)

$\therefore \Delta AMB\ |||\ \Delta PBC$ (Equiangular)

ii) $\frac{BC}{MB} = \frac{CP}{BA}$ (Corresponding sides in similar triangles are in same ratio.) (1)

$\frac{5}{MB} = \frac{CP}{5}$

But $MB^2 = 5^2 + \left(\frac{5}{2}\right)^2$ (Pythagoras)

$MB = 5\sqrt{5}/2$ (1)

$\therefore CP = \frac{25\times 2}{5\sqrt{5}} = 2\sqrt{5}$ cm

Drop ⊥ from P to N on CD.

$\tan\alpha = \frac{BP}{CP} = \frac{1}{2}$.

$\therefore \sin\alpha = 1/\sqrt{5}$.

$\therefore CN = 2\sqrt{5}\sin\alpha = 2$

$\therefore DN = 3$ (1)

$PN = 2\sqrt{5}\cos\alpha = 2\sqrt{5}\,\frac{2}{\sqrt{5}} = 4$

$\therefore PD^2 = PN^2 + DN^2$

$PD = 5 \quad (= DC)$ (1)

$\therefore \Delta PDC$ is isosceles. (PD = DC) (2 equal sides) (1)

Many methods